

# Higgsless Electroweak Theory following from the Spherical Geometry

N.A. Gromov

Department of Mathematics,  
Komi Science Center UrD RAS

Kommunisticheskaya st., 24, Syktyvkar, 167982, Russia

E-mail: gromov@dm.komisc.ru

## Abstract

A new formulation of the Electroweak Model with 3-dimensional spherical geometry in the target space is suggested. The free Lagrangian in the spherical field space along with the standard gauge field Lagrangian form the full Higgsless Lagrangian of the model, whose second order terms reproduce the same fields with the same masses as the Standard Electroweak Model. The vector bosons and electron masses are generated automatically, so there is no need in special mechanism.

PACS 12.15–y.

## 1 Introduction

The Standard Electroweak Model (SEWM) based on gauge group  $SU(2) \times U(1)$  gives a good description of electroweak processes. One of the unsolved problems is the origin of electroweak symmetry breaking. In the standard formulation the scalar field (Higgs boson) performs this task via Higgs mechanism, which generates a mass terms for vector bosons. However, it is not yet experimentally verified whether electroweak symmetry is broken by such a Higgs mechanism, or by something else.

The emergence of large number Higgsless models [2]–[10] was stimulated by difficulties with Higgs boson. These models are mainly based on extra dimensions of different types or larger gauge groups. The construction given in [11] is based on an observation: the underlying group of SEWM can be represented as a semidirect product of  $U(1)$  and  $SU(2)$ .

In the previous papers [12],[13], where the gauge field theories based on non-semisimple contracted Cayley-Klein groups were considered, it was noted that Higgs mechanism looks very artificial and Higgs boson being its artefact is unobservable. In the present paper a new formulation of the Higgsless Electroweak Model with the 3-dimensional spherical geometry in the target space is suggested.

## 2 Standard Electroweak Model

The bosonic sector of SEWM is  $SU(2) \times U(1)$  gauge theory in the space  $\Phi_2(\mathbf{C})$  of fundamental representation of  $SU(2)$ . The bosonic Lagrangian is given by the sum

$$L_B = L_A + L_\phi, \tag{1}$$

where

$$L_A = \frac{1}{8g^2} \text{Tr}(F_{\mu\nu})^2 - \frac{1}{4}(B_{\mu\nu})^2 = -\frac{1}{4}[(F_{\mu\nu}^1)^2 + (F_{\mu\nu}^2)^2 + (F_{\mu\nu}^3)^2] - \frac{1}{4}(B_{\mu\nu})^2 \quad (2)$$

is the gauge field Lagrangian for  $SU(2) \times U(1)$  group and

$$L_\phi = \frac{1}{2}(D_\mu\phi)^\dagger D_\mu\phi - V(\phi) \quad (3)$$

is the matter field Lagrangian. Here  $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \in \Phi_2(\mathbf{C})$ ,  $D_\mu$  are the covariant derivatives

$$D_\mu\phi = \partial_\mu\phi - ig \left( \sum_{k=1}^3 T_k A_\mu^k \right) \phi - ig' Y B_\mu \phi, \quad (4)$$

where  $T_k = \frac{1}{2}\sigma_k$ , with  $\sigma_k$  being Pauli matrices, are generators of  $SU(2)$  and  $Y = \frac{1}{2}\mathbf{1}$  is generator of  $U(1)$ . The gauge fields

$$A_\mu(x) = -ig \sum_{k=1}^3 T_k A_\mu^k(x), \quad B_\mu(x) = -ig' Y B_\mu(x) \quad (5)$$

take their values in Lie algebras  $su(2)$ ,  $u(1)$  respectively, and the stress tensors are

$$F_{\mu\nu}(x) = \mathcal{F}_{\mu\nu}(x) + [A_\mu(x), A_\nu(x)], \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (6)$$

The potential  $V(\phi)$  in (3) is introduced by hand in a special form

$$V(\phi) = \frac{\lambda}{4} (\phi^\dagger \phi - v^2)^2, \quad (7)$$

where  $\lambda, v$  are constants.

The Lagrangian  $L_B$  (1) describe massless fields. To generate mass terms for the vector bosons without breaking the gauge invariance one uses the Higgs mechanism. One of  $L_B$  ground states

$$\phi^{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad A_\mu^k = B_\mu = 0 \quad (8)$$

is taken as a vacuum state of the model, and small field excitations

$$\phi_1(x), \quad \phi_2(x) = v + \chi(x), \quad A_\mu^a(x), \quad B_\mu(x) \quad (9)$$

with respect to the vacuum are regarded. The matrix  $Q = Y + T^3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ , which annihilates the ground state  $Q\phi^{vac} = 0$ , is the generator of the electromagnetic subgroup  $U(1)_{em}$ . The new fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (A_\mu^1 \mp iA_\mu^2), \quad Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (gA_\mu^3 - g'B_\mu), \quad A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g'A_\mu^3 + gB_\mu) \quad (10)$$

are introduced, where  $W_\mu^\pm$  are complex  $(W_\mu^-)^* = W_\mu^+$  and  $Z_\mu, A_\mu$  are real.

The second order terms of the Lagrangian (1) are as follows

$$\begin{aligned} L_B^{(2)} = & -\frac{1}{2}\mathcal{W}_{\mu\nu}^+\mathcal{W}_{\mu\nu}^- + m_W^2 W_\mu^+ W_\mu^- - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu} - \\ & -\frac{1}{4}\mathcal{Z}_{\mu\nu}\mathcal{Z}_{\mu\nu} + \frac{1}{2}m_Z^2 Z_\mu Z_\mu + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m_\chi^2\chi^2, \end{aligned} \quad (11)$$

where  $\mathcal{Z}_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ ,  $\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ,  $\mathcal{W}_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$ . This describes massive vector fields  $W_\mu^\pm$  with identical mass  $m_W = \frac{1}{2}gv$  ( $W$ -bosons), massless vector field  $A_\mu$ ,  $m_A = 0$  (photon), massive vector field  $Z_\mu$  with the mass  $m_Z = \frac{v}{2}\sqrt{g^2 + g'^2}$  ( $Z$ -boson), and massive scalar field  $\chi$ ,  $m_\chi = \sqrt{2}\lambda v$  (Higgs boson).

$W$ - and  $Z$ -bosons have been observed and have the masses  $m_W = 80\text{GeV}$ ,  $m_Z = 91\text{GeV}$ . Higgs boson has not been experimentally verified up to now.

### 3 Higgsless Electroweak Model with 3D spherical matter space

The complex 2D space  $\Phi_2$  can be regarded as 4D real Euclidean space  $\mathbf{R}_4$ . Let us introduce the real fields  $r, \bar{\psi} = (\psi_1, \psi_2, \psi_3)$  by

$$\phi_1 = r(\psi_2 + i\psi_1), \quad \phi_2 = r(1 + i\psi_3). \quad (12)$$

It is easy to see that the quadratic form  $\phi^\dagger\phi = \phi_1^*\phi_1 + \phi_2^*\phi_2 = R^2$  is invariant with respect to gauge transformations. For the real fields this form is written as  $r^2(1 + \bar{\psi}^2) = R^2$ , where  $\bar{\psi}^2 = \psi_1^2 + \psi_2^2 + \psi_3^2$ , therefore

$$r = \frac{R}{\sqrt{1 + \bar{\psi}^2}} \quad (13)$$

Hence there are three independent real fields  $\bar{\psi}$ . These fields belong to the space  $\Psi_3$  with noneuclidean spherical geometry which is realized on the 3D sphere in 4D Euclidean space  $\mathbf{R}_4$ . The fields  $\bar{\psi}$  are intrinsic Beltrami coordinates on  $\Psi_3$ .

The potential (7) is the constant  $V(\phi) = \lambda(R^2 - v^2)^2/4$  and  $V(\phi) = 0$  for  $R = v$ . Therefore, let us define the *free* gauge invariant matter field Lagrangian  $L_\psi$  with the help of the metric tensor

$$g_{kk}(\bar{\psi}) = \frac{1 + \bar{\psi}^2 - \psi_k^2}{(1 + \bar{\psi}^2)^2}, \quad g_{kl}(\bar{\psi}) = \frac{-\psi_k\psi_l}{(1 + \bar{\psi}^2)^2} \quad (14)$$

of  $\Psi_3$  in the form

$$L_\psi = \frac{R^2}{2} \sum_{k,l=1}^3 g_{kl} D_\mu \psi_k D_\mu \psi_l = \frac{R^2 \left[ (1 + \bar{\psi}^2)(D_\mu \bar{\psi})^2 - (\bar{\psi}, D_\mu \bar{\psi})^2 \right]}{2(1 + \bar{\psi}^2)^2}. \quad (15)$$

The covariant derivatives (4) are obtained using the representations of generators for the algebras  $su(2)$ ,  $u(1)$  in the space  $\Psi_3$  [15]

$$\begin{aligned} T_1 \bar{\psi} &= \frac{i}{2} \begin{pmatrix} -(1 + \psi_1^2) \\ \psi_3 - \psi_1 \psi_2 \\ -(\psi_2 + \psi_1 \psi_3) \end{pmatrix}, & T_2 \bar{\psi} &= \frac{i}{2} \begin{pmatrix} -(\psi_3 + \psi_1 \psi_2) \\ -(1 + \psi_2^2) \\ \psi_1 - \psi_2 \psi_3 \end{pmatrix}, \\ T_3 \bar{\psi} &= \frac{i}{2} \begin{pmatrix} -\psi_2 + \psi_1 \psi_3 \\ \psi_1 + \psi_2 \psi_3 \\ 1 + \psi_3^2 \end{pmatrix}, & Y \bar{\psi} &= \frac{i}{2} \begin{pmatrix} -(\psi_2 + \psi_1 \psi_3) \\ \psi_1 - \psi_2 \psi_3 \\ -(1 + \psi_3^2) \end{pmatrix}, \end{aligned} \quad (16)$$

and are as follows:

$$\begin{aligned} D_\mu \psi_1 &= \partial_\mu \psi_1 + \frac{g}{2} \left[ -(1 + \psi_1^2) A_\mu^1 - (\psi_3 + \psi_1 \psi_2) A_\mu^2 - (\psi_2 - \psi_1 \psi_3) A_\mu^3 \right] - \frac{g'}{2} (\psi_2 + \psi_1 \psi_3) B_\mu, \\ D_\mu \psi_2 &= \partial_\mu \psi_2 + \frac{g}{2} \left[ (\psi_3 - \psi_1 \psi_2) A_\mu^1 - (1 + \psi_2^2) A_\mu^2 + (\psi_1 + \psi_2 \psi_3) A_\mu^3 \right] + \frac{g'}{2} (\psi_1 - \psi_2 \psi_3) B_\mu, \\ D_\mu \psi_3 &= \partial_\mu \psi_3 + \frac{g}{2} \left[ -(\psi_2 + \psi_1 \psi_3) A_\mu^1 + (\psi_1 - \psi_2 \psi_3) A_\mu^2 + (1 + \psi_3^2) A_\mu^3 \right] - \frac{g'}{2} (1 + \psi_3^2) B_\mu. \end{aligned} \quad (17)$$

The gauge fields Lagrangian does not depend on the fields  $\phi$  and therefore remains unchanged (2).

For small fields, the second order Lagrangian (15) is written as

$$L_\psi^{(2)} = \frac{R^2}{2} \left[ (D_\mu \bar{\psi})^{(1)} \right]^2 = \frac{R^2}{2} \sum_{k=1}^3 \left[ (D_\mu \psi_k)^{(1)} \right]^2, \quad (18)$$

where linear terms in covariant derivatives (17) have the form

$$\begin{aligned} (D_\mu \psi_1)^{(1)} &= \partial_\mu \psi_1 - \frac{g}{2} A_\mu^1 = -\frac{g}{2} \left( A_\mu^1 - \frac{2}{g} \partial_\mu \psi_1 \right) = -\frac{g}{2} \hat{A}_\mu^1, \\ (D_\mu \psi_2)^{(1)} &= \partial_\mu \psi_2 - \frac{g}{2} A_\mu^2 = -\frac{g}{2} \left( A_\mu^2 - \frac{2}{g} \partial_\mu \psi_2 \right) = -\frac{g}{2} \hat{A}_\mu^2, \\ (D_\mu \psi_3)^{(1)} &= \partial_\mu \psi_3 + \frac{g}{2} A_\mu^3 - \frac{g'}{2} B_\mu = \partial_\mu \psi_3 + \frac{1}{2} (g A_\mu^3 - g' B_\mu) = \frac{1}{2} \sqrt{g^2 + g'^2} Z_\mu. \end{aligned} \quad (19)$$

For the new fields

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} \left( \hat{A}_\mu^1 \mp i \hat{A}_\mu^2 \right), & (W_\mu^-)^* &= W_\mu^+ \\ Z_\mu &= \frac{g A_\mu^3 - g' B_\mu + 2 \partial_\mu \psi_3}{\sqrt{g^2 + g'^2}}, & A_\mu &= \frac{g' A_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}} \end{aligned} \quad (20)$$

Lagrangian (18) is rewritten as follows

$$L_\psi^{(2)} = + \frac{R^2 g^2}{4} W_\mu^+ W_\mu^- + \frac{R^2 (g^2 + g'^2)}{8} (Z_\mu)^2. \quad (21)$$

The quadratic part of the full Lagrangian

$$L_B^{(2)} = -\frac{1}{2}\mathcal{W}_{\mu\nu}^+\mathcal{W}_{\mu\nu}^- + m_W^2 W_\mu^+ W_\mu^- - \frac{1}{4}(\mathcal{F}_{\mu\nu})^2 - \frac{1}{4}(\mathcal{Z}_{\mu\nu})^2 + \frac{m_Z^2}{2}(Z_\mu)^2, \quad (22)$$

where

$$m_W = \frac{Rg}{2}, \quad m_Z = \frac{R}{2}\sqrt{g^2 + g'^2}, \quad (23)$$

and  $\mathcal{Z}_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ ,  $\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ,  $\mathcal{W}_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$ , describes all the experimentally verified parts of SEWM but does not include the scalar Higgs field. For  $R = v$  the masses (23) are identical to those of the SEWM (11).

The fermion Lagrangian of SEWM is taken in the form [14]

$$L_F = L_l^\dagger i\tilde{\sigma}_\mu D_\mu L_l + e_r^\dagger i\sigma_\mu D_\mu e_r - h_e[e_r^\dagger(\phi^\dagger L_l) + (L_l^\dagger \phi)e_r], \quad (24)$$

where  $L_l = \begin{pmatrix} \nu_{e,l} \\ e_l^- \end{pmatrix}$  is the  $SU(2)$ -doublet,  $e_l^-$  the  $SU(2)$ -singlet,  $h_e$  is constant and  $e_r, e_l, \nu_e$  are two component Lorentzian spinors. Here  $\sigma_\mu$  are Pauli matrices,  $\sigma_0 = \tilde{\sigma}_0 = \mathbf{1}$ ,  $\tilde{\sigma}_k = -\sigma_k$ . The covariant derivatives  $D_\mu L_l$  are given by (4) with  $L_l$  instead of  $\phi$  and  $D_\mu e_r = (\partial_\mu + ig'B_\mu)e_r$ . The convolution on the inner indices of  $SU(2)$ -doublet is denoted by  $(\phi^\dagger L_l)$ .

The matter fields  $\phi$  appear in Lagrangian (24) only in mass terms. With the use of (12) and (13), these mass terms are rewritten in the form

$$\begin{aligned} h_e[e_r^\dagger(\phi^\dagger L_l) + (L_l^\dagger \phi)e_r] &= \frac{h_e R}{\sqrt{1 + \bar{\psi}^2}} \left[ e_r^\dagger e_l^- + e_l^{-\dagger} e_r + \right. \\ &\quad \left. + i\psi_3 (e_l^{-\dagger} e_r - e_r^\dagger e_l^-) + i\psi_1 (\nu_{e,l}^\dagger e_r - e_r^\dagger \nu_{e,l}) + i\psi_2 (\nu_{e,l}^\dagger e_r + e_r^\dagger \nu_{e,l}) \right]. \end{aligned} \quad (25)$$

Its second order terms  $h_e R (e_r^\dagger e_l^- + e_l^{-\dagger} e_r)$  provide the electron mass  $m_e = h_e R$ , and neutrino remain massless.

## 4 Conclusion

The suggested formulation of the Electroweak Model with the gauge group  $SU(2) \times U(1)$  based on the 3-dimensional spherical geometry in the target space describes all experimentally observed fields, and does not include the (up to now unobserved) scalar Higgs field. The *free* Lagrangian in the spherical matter field space is used instead of Lagrangian (3) with the potential (7) of the special form (sombbrero). The gauge field Lagrangian is the standard one. There is no need in Higgs mechanism since the vector field masses are generated automatically.

The motion group of the spherical space  $\Psi_3$  is isomorphic to  $SO(4)$ . It is known [1], that the bosonic sector of SEWM has the custodial  $SU(2)$ -symmetry, and potential (7) is globally invariant with respect of  $SO(4)$  which is locally isomorphic to the group  $SU(2)_L \times SU(2)_R$ , where  $SU(2)_L$  is the gauge group of SEWM. The appearance of the extra  $SU(2)_R$ -symmetry is interpreted as transformation of  $SU(2)$ -doublet by pseudoreal representation, that is the complex conjugate doublet is equivalent to the initial one. The introduction of the real fields  $\bar{\psi}$  is connected in a certain sense with  $SO(4)$ -symmetry.

## 5 Acknowledgments

The author is grateful to A.V. Zhubr and V.V. Kuratov for helpful discussions.

## References

- [1] C. Grojean, Usp. Fiz. Nauk 177 (2007) 3 (in Russian).
- [2] E.H. Simmons, R.S. Chuvukula, H.-J. He, M. Kurachi, M. Tanabashi, hep-ph/0606019 v1.
- [3] R. Casalbuoni, hep-ph/0509165 v1.
- [4] R.S. Chivukala, E.H. Simmons, H.-J. He, M. Kurachi, M. Tanabashi, hep-ph/0406073 v3.
- [5] A.A. Slavnov, hep-th/0601125 v1.
- [6] A.A. Slavnov, hep-th/0604052 v1.
- [7] G. Cacciapaglia, C. Csáki, G. Marandella, J. Terning, hep-ph/0607156 v1.
- [8] C. Csáki, J. Hubisz, P. Meade, hep-ph/0510275 v1.
- [9] C. Csáki, hep-ph/0412339 v1.
- [10] C. Csáki, C. Grojean, L. Pilo, J. Terning, hep-ph/0308038 v2.
- [11] A. Sevestyanov, hep-th/0702149 v1.
- [12] N.A. Gromov, hep-th/0611079.
- [13] N.A. Gromov, hep-th/0611092.
- [14] V.A. Rubakov, Classical Gauge Fields, Editorial URSS, Moscow, 1999 (in Russian).
- [15] N.A. Gromov, Spherical Space permits to go without Higgs Boson, Komi Science Center RAS, Syktyvkar, 2007 ("Scientific Reports", N. 493 ) (in Russian).